MATH 158 PF

Algebra Review -- Addition of Polynomial Fractions

This 1/2 unit module covers addition and subtraction of polynomial fractions. It is assumed that the student understands the basic factoring of polynomials. A brief review of the least common multiple of two or more polynomials is included.

To earn 1/2 unit of credit, a student must study the entire module and work all exercises. The last exercise set, 91-105, features a collection of the different problems from the previous sets of exercises. A student must submit to the math lab instructor a notebook showing full details of having completed all these exercises from the last problem set. Once the instructor verifies that all these exercises have been worked correctly and with sufficient detail, a student will be given an examination on complete factoring.

If the examination is completed with suitable accuracy, credit will be awarded. Alternate examinations will be available if the accuracy of the first one is unsatisfactory.

Mark Eastman, April 1987

MATH 58 - Adding Polynomial Fractions

A. Objective

The objective of this module is the ability to add and subtract polynomial fractions, and to express answers in lowest terms at the level of elementary algebra.

B. Review of preliminary concepts

1. A Polynomial Fraction is an expression of the form p/q, where p and q are polynomials, and q is not the zero polynomial. This type of expression can be viewed as an indicated division, p/q. The two polynomial fractions are equal in case their cross-products are the same, that is p/q = r/s in case p*s = q*r.

Polynomial fraction is in lowest terms in case the polynomials in the numerator and the denominator have no common factors.

Examples:

a. $\dfrac{x}{x + 4}$ is in lowest terms

b. $\dfrac{x^2 - 1}{x^2 + x - 6} = \dfrac{(x + 1)(x - 1)}{(x + 3)(x - 2)}$ is in lowest terms.

c. $\dfrac{2x + 4}{6x - 10}$ $\dfrac{2(x + 2)}{2(3x - 5)}$

is not in lowest terms as the numerator and the denominator have the common factor 2.

d. $\dfrac{x^2 - 9}{x^2 + 2x - 15} = \dfrac{(x + 3)(x - 3)}{(x + 5)(x - 3)}$

is not in lowest terms as the numerator and the denominator have the common factor (x -3).

2. A polynomial fraction can be put in lowest terms by factoring both polynomials and cancelling in pairs those factors which appear both in the numerator and

the denominator. If numbers appear as <u>factors</u> in both the numerator and the denominator, these number factors should be factored into primes to determine which, if any factors should be cancelled. Note also that it may be necessary to factor out a (−1) from a polynomial in order to get common factors that cancel.

Examples:

a. $$\frac{x^2 + 3x}{x^2 + 2x - 3} = \frac{x(x + 3)}{(x + 3)(x - 1)} = \frac{x}{x - 1}$$

b. $$\frac{6x - 18}{10x + 40} = \frac{6(x - 3)}{10(x + 4)} = \frac{2 * 3(x - 3)}{2 * 5(x - 4)} = \frac{3(x - 3)}{5(x - 4)}$$

c. $$\frac{2x - x^2}{x^2 - 4} = \frac{-1(x^2 - 2x)}{(x + 2)(x - 2)} = \frac{-1 * x(x - 2)}{(x + 2)(x - 2)} = \frac{-x}{x + 2}$$

In the last example, a (−1) was factored out of the polynomial in the numerator, so that the coefficient of the x − term was positive. Note that more than one of these techniques may be necessary when reducing a polynomial fraction.

3. <u>Building fractions</u> is necessary for addition and subtraction of polynomial fractions. An equal fraction is obtained when both the numerator and the denominator of a polynomial fraction are multiplied by the same factor(s).

Examples:

a. $\frac{3x}{2y}$ can be built up by multiplying both the numerator and the denominator by 5x :

$$\frac{3x}{2y} = \frac{3x * 5x}{2y * 5x} = \frac{15x^2}{10x\ y}$$

b. $\frac{x - 4}{x - 5}$

can be built up by a factor of $x + 3$:

$$\frac{x - 4}{x + 5} = \frac{(x - 4)(x + 3)}{(x + 5)(x + 3)} = \frac{x^2 - x - 12}{(x + 5)(x + 3)}$$

c. $\dfrac{2(x - 7)}{3x - 12}$

can be built up by a factor of $2(x + 1)$:

$$\frac{2(x - 7)}{3x^2 - 12} = \frac{2(x - 7)}{3(x^2 - 4)} = \frac{2(x - 7)}{3(x + 2)(x - 2)}$$

$$= \frac{2(x - 7) * [2(x + 1)]}{3(x + 2)(x - 2)[2(x + 2)]} = \frac{4(x^2 - 6x - 7)}{6(x + 2)(x - 2)(x + 1)}$$

$$= \frac{4x^2 - 24x - 28}{6(x + 2)(x - 2)(x + 1)}$$

4. The <u>least</u> <u>common</u> <u>multiple</u> or <u>LCM</u>, of two or more polynomials is the polynomial of least degree and smallest coefficients such that it is a multiple of each polynomial given. To obtain the LCM of two or more polynomials, first factor each polynomial completely. If the polynomials have numbers as factors, write the prime factorization of these number factors. The LCM will be the product of the least number of factors so that the factors of each polynomial appear in this product. One can obtain the LCM by writing all the factors of the first polynomial, and then multiplying this product by those factors in the remaining lists which are not present.

Examples:

a. LCM $(10x^2 y, 15xy^2)$.

$$10x^2 y = 2 * 5 * x * x * y.$$
$$15xy^2 = 3 * 5 * x * y * y.$$

$$\text{LCM} = \underbrace{2 * 5 * x * x * y}_{\text{factors of first polynomial}} * \underbrace{3 * y}_{\text{factors of second polynomial not yet in product.}} = 30x^2 * y^2 .$$

b. LCM $(x^2 - 4, x^2 + 3x - 10)$.

$$x^2 - 4 = (x + 2)(x - 2)$$
$$x^2 + 3x - 10 = (x + 5)(x - 2)$$

$$\text{LCM} = \underbrace{(x + 2)(x - 2)}_{\text{from first polynimial}}\underbrace{(x + 5)}_{\text{from second polynomial}}$$

Note that the LCM will be left in factored form (except the monomial factors).

c. LCM $(2x^2 + 4x - 30,\ x^2 + 8x + 15,\ 6x^2 - 54)$

$$2x^2 + 4x - 30 = 2(x^2 + 2x - 15) = 2(x + 5)(x - 3)$$
$$x^2 + 8x + 15 = (x + 5)(x + 3)$$
$$6x^2 - 54 = 6(x^2 - 9) = 2 * 3\ (x + 3)(x - 3)$$

$$\text{LCM} = \underbrace{2(x + 5)(x - 3)}_{\text{first polynomial}}\underbrace{(x + 3)}_{\text{second polynomial}} * \underbrace{3}_{\text{third polynomial}} = 6\ (x + 5)(x - 3)(x + 3)$$

It may be necessary to factor a (−1) out of the coefficients in order to find the LCM. A good rule to follow is to factor out the proper coefficient so that the remaining coefficient of the largest degree term is positive.

Example:

a. LCM $(8x - 12,\ 6 - 4x)$

$$8x - 12 = 4(2x - 3) = 2 * 2(2x - 3)$$
$$6 - 4x = -1(4x - 6) = -1 * 2(2x - 3)$$

$$\text{LCM} = 2 * 2(2x - 3) * (-1) = -4(2x - 3)$$

b. LCM $(x^2 + x - 20,\ 4x - x^2)$

$$x^2 + x - 20 = (x + 5)(x - 4)$$
$$4x - x^2 = -1(x^2 - 4x) = -1 * x\ (x - 4)$$

$$\text{LCM} = -1 * x\ (x - 4)(x + 5).$$

C. Addition of polynomial fractions

When adding numerical fractions, it is required to express all the fractions in the sum with the same denominator. This is most efficiently done by finding the <u>Least Common Denominator</u> or LCD of the fractions, which is the least common multiple of the denominators of the fractions. Each fraction is then built up to a fraciton with the LCD as the denominator. The numerators are then added together, and the resulting sum is reduced to lowest terms:

Example: $\dfrac{7}{10} + \dfrac{2}{15}$

Step 1: LCM (10, 15) = 2 * 5 * 3 = 30.

Step 2: $\dfrac{7}{10} = \dfrac{7 * 3}{10 * 3} = \dfrac{21}{30}$

$\dfrac{2}{15} = \dfrac{2 * 2}{15 * 2} = \dfrac{4}{30}$

Step 3: $\dfrac{7}{10} + \dfrac{2}{15} = \dfrac{21}{30} + \dfrac{4}{30} = \dfrac{25}{30}$

Step 4: $\dfrac{25}{30} = \dfrac{5 * 5}{5 * 6} = \dfrac{5}{6}$

This method can be used for adding polynomial fractions. If all the denominators of the fractions in the sum are the same, the numerators are added together and put over the common denominator. If the denominators are not the same, the following method is used:

1. Find the LCD for the fractions.

2. Build up each fraction by the appropriate factors so that each fraction has the LCD in the denominator; leave the denominator in factored form.

3. Do the indicated operations in each numerator.

4. Write the sum of the numerators over the LCD.

5. Simplify the numerator of the result by combining similar terms.

6. Factor the numerator, if possible, and reduce to lowest terms. If the numerator cannot be factored, then the sum is already in lowest terms.

D. Exercises

In this exercise set, the denominators of the polynomial fractions are the same. To add these fractions, just add the numerators together and place the result over the common denominator.

Worked example:

$$\frac{2x + 3}{x + 1} + \frac{x - 2}{x + 1} = \frac{(2x + 3) + (x - 2)}{x + 1} = \frac{3x + 1}{x + 1}$$

****

1. $\frac{x + 1}{x} + \frac{3}{x}$

2. $\frac{x - 5}{x - 4} + \frac{x + 2}{x - 4}$

3. $\frac{x - 3}{2x + 5} + \frac{3x + 8}{2x + 5}$

4. $\frac{2x}{3x + 1} + \frac{3x - 4}{3x + 1}$

5. $\frac{x - 5}{x^2 + 3x} + \frac{3x + 2}{x^2 + 3x}$

In this exercise set, the denominators are different, but are monomials (a single term). Find the LCD; build up each fraction so that the denominator is the LCD; and add the numerators togegether.

Worked example:

$$\frac{x + 3}{6x} + \frac{2y - 7}{15x^2 y} \qquad \begin{aligned} 6x &= 2 * 3 * x * x * x \\ 15x^2 y &= 3 * 5 * x * x * y \end{aligned}$$

$$LCD = 2 * 3 * 5 * x * x * x * y \;=\; 30x\ y$$

$$\frac{x + 3}{6x} + \frac{2y - 7}{15x\ y} = \frac{(x + 3) * 5y}{6x\ * 5y} + \frac{(2y - 7) * 2x}{15x\ y * 2x}$$

$$= \frac{5xy + 15y}{30x\ y} + \frac{4xy - 14x}{30x\ y} = \frac{(5xy + 15y) + (4xy - 14x)}{30x\ y}$$

$$= \frac{9xy + 15y - 14x}{30x\ y}$$

****

6. $\dfrac{x + 4}{x} + \dfrac{2x - 3}{x^2}$

7. $\dfrac{x + 4}{2x} + \dfrac{3y - 5}{6xy}$

8. $\dfrac{x + 3}{2x} + \dfrac{2x - 3}{5x^2}$

9. $\dfrac{x + 3y}{6x^2} + \dfrac{2x - 5y}{4xy}$

10. $\dfrac{2x - 3y}{5xy} + \dfrac{3x - 7y}{5x^2}$

In this exercise set, the LCD is a first degree polynomial. Be sure to express any integer factor of a denominator as a product of primes.

<u>Worked example:</u>

$$\frac{x + 1}{6x - 12} + \frac{3x}{4x - 8}$$

$6x - 12 = 6(x - 2) = 2 * 3(x - 2)$
$4x - 8 = 4(x - 2) = 2 * 2(x - 2)$
$LCD = 2 * 2 * 3(x - 2) = 12(x - 2)$

$$\frac{x + 1}{6x - 12} + \frac{3x}{4x - 8} = \frac{x + 1}{6(x - 2)} + \frac{3x}{4(x - 2)}$$

$$= \frac{(x + 1) * 2}{6(x - 2) * 2} + \frac{3x * 3}{4(x - 2) * 3}$$

$$= \frac{2x + 2}{12(x - 2)} + \frac{9x}{12(x - 2)}$$

$$= \frac{(2x + 2) + 9x}{12(x - 2)}$$

$$= \frac{11x + 2}{12(x - 2)}$$

****

11. $\dfrac{x + 3}{4} + \dfrac{3x - 5}{8x - 16}$

12. $\dfrac{x - 2}{2x + 6} + \dfrac{2x + 1}{5x + 15}$

13. $\dfrac{x + 3}{2x + 5} + \dfrac{x - 4}{6}$

14. $\dfrac{2x + 7}{12x + 4} + \dfrac{x - 5}{3x + 1}$

15. $\dfrac{x + 4}{6x - 15} + \dfrac{2x - 7}{8x - 20}$

In this exercise set, one of the denominators will be the LCD. After factoring this denominator, build up the other fraction and add.

Worked example:

$$\frac{3x - 5}{2x^2 + 7x + 3} + \frac{x + 4}{2x + 1} = \frac{3x - 5}{(2x + 1)(x + 3)} + \frac{x + 4}{2x + 1}$$

$$= \frac{3x - 5}{(2x + 1)(x + 3)} + \frac{(x + 4)(x + 3)}{(2x + 1)(x + 3)}$$

$$= \frac{3x - 5}{(2x + 1)(x + 3)} + \frac{x^2 + 7x + 12}{(2x + 1)(x + 3)}$$

$$= \frac{(3x - 5) + (x^2 + 7x + 12)}{(2x + 1)(x + 3)}$$

$$= \frac{x^2 + 10x + 7}{(2x + 1)(x + 3)}$$

****

16. $\dfrac{x}{x - 2} + \dfrac{3}{x^2 - 4}$

17. $\dfrac{x - 1}{x + 3} + \dfrac{2x + 5}{x^2 + x - 6}$

18. $\dfrac{x + 3}{x^2 - 4x - 5} + \dfrac{x - 1}{x - 5}$

19. $\dfrac{x}{x + 3} + \dfrac{2x + 1}{x^2 + 6x + 9}$

20. $\dfrac{x + 1}{x^2 - 3x - 10} + \dfrac{x}{x + 2}$

21. $\dfrac{3x + 4}{2x^2 - 7x - 15} + \dfrac{x + 2}{x - 5}$

22. $\dfrac{2x - 5}{3x - 7} + \dfrac{5x + 3}{3x^2 - x - 14}$

23. $\dfrac{2x + 7}{6x^2 + 17x - 3} + \dfrac{5}{x - 3}$

24. $\dfrac{3x + 8}{6x^2 + 7x - 20} + \dfrac{x + 3}{2x + 5}$

25. $\dfrac{2x - 3}{5x - 7} + \dfrac{3x - 5}{10x^2 + x - 21}$

In these exercises, both fractions must be built up. The LCD is the product of the two denominators.

<u>Worked example:</u>

$$\frac{x + 3}{2x + 5} + \frac{4}{x - 3} = \frac{(x + 3)(x - 3)}{(2x + 5)(x - 3)} + \frac{4\ (2x + 5)}{(x - 3)(2x + 5)}$$

$$= \frac{x^2 - 9}{(2x + 5)(x - 3)} + \frac{8x + 20}{(x - 3)(2x + 5)}$$

$$= \frac{(x^2 - 9) + (8x + 20)}{(2x + 5)(x - 3)}$$

$$= \frac{x + 8x + 11}{(2x + 5)(x - 3)}$$

****

26. $\dfrac{x}{x + 5} + \dfrac{3}{x - 7}$

27. $\dfrac{4}{x + 2} + \dfrac{3}{2x - 3}$

28. $\dfrac{x + 4}{3x + 1} + \dfrac{1}{x - 3}$

29. $\dfrac{2x + 4}{x + 3} + \dfrac{x - 1}{2x + 7}$

30. $\dfrac{4}{(x - 6)} + \dfrac{x - 5}{3x + 2}$

31. $\dfrac{x}{2x + 3} + \dfrac{7}{2x - 5}$

32. $\dfrac{2x + 1}{3x - 4} + \dfrac{x}{2x + 1}$

33. $\dfrac{5}{2x - 7} + \dfrac{3x}{4x + 3}$

34. $\dfrac{2x + 3}{3x + 1} + \dfrac{x + 5}{3x - 2}$

35. $\dfrac{x + 3}{6x + 1} + \dfrac{2}{2x - 5}$

In this exercise set, three polynomial fractions are added. The LCD will be a second degree polynomial, that is a product of two first degree binomials.

Worked example:

$$\frac{x}{2x + 1} + \frac{x + 3}{x - 2} + \frac{3x - 5}{2x^2 - 3x - 2}$$

$$= \frac{x}{2x + 1} + \frac{x + 3}{x - 2} + \frac{3x - 5}{(2x + 1)(x - 2)}$$

$$= \frac{x(x - 2)}{(2x + 1)(x - 2)} + \frac{(x + 3)(2x + 1)}{(x - 2)(2x + 1)} + \frac{(3x - 5)}{(2x + 1)(x - 2)}$$

$$= \frac{(x^2 - 2x) + (x^2 + 7x + 3) + (3x - 5)}{(2x + 1)(x - 2)}$$

$$= \frac{2x^2 + 8x - 2}{(2x + 1)(x - 2)}$$

$$= \frac{2(x^2 + 4x - 1)}{(2x + 1)(x - 2)}$$

****

36. $\dfrac{x}{x + 8} + \dfrac{2x - 7}{x^2 + 5x - 24} + \dfrac{x + 4}{x - 3}$

37. $\dfrac{3x + 8}{2x^2 - 9x - 18} + \dfrac{x + 5}{2x + 3} + \dfrac{2x - 1}{x - 6}$

38. $\dfrac{x + 2}{x + 4} + \dfrac{4x + 9}{3x^2 + 7x - 20} + \dfrac{4}{3x - 5}$

39. $\dfrac{3x + 2}{2x + 5} + \dfrac{x + 3}{2x - 5} + \dfrac{3x + 7}{4x^2 - 25}$

40. $\dfrac{x + 4}{3x - 7} + \dfrac{2x - 1}{2x + 5} + \dfrac{4x + 7}{6x^2 + x - 35}$

The following exercises involve addition problems where the numerator of the sum can be factored, and the numerator and the denominator will have a common factor, so that the resulting sum can be reduced.

<u>Worked example:</u>

$$\frac{x}{x^2 - 9} + \frac{x - 4}{x - 4x + 3}$$

$$= \frac{x}{(x + 3)(x - 3)} + \frac{x - 4}{(x - 3)(x - 1)}$$

LCD is $(x + 3)(x - 3)(x - 1)$

$$= \frac{x\ (x - 1)}{(x + 3)(x - 3)(x - 1)} + \frac{(x - 4)(x + 3)}{(x - 3)(x + 1)(x + 3)}$$

$$= \frac{(x^2 - x) + (x^2 - x - 12)}{(x + 3)(x - 3)(x - 1)}$$

$$= \frac{2x^2 - 2x - 12}{(x + 3)(x - 3)(x - 1)}$$

$$= \frac{2\ (x^2 - x - 6)}{(x + 3)(x - 3)(x - 1)}$$

$$= \frac{2(x + 2)(x - 3)}{(x + 3)(x - 3)(x - 1)} = \frac{2(x + 2)}{(x + 3)(x - 1)}$$

****

41. $\dfrac{1}{x^2 + 5x + 6} + \dfrac{2}{x^2 + 8x + 15}$

42. $\dfrac{4}{x^2 + 2x - 3} + \dfrac{3}{x^2 + 5x + 4}$

43. $\dfrac{2}{x^2 - 6x + 8} + \dfrac{5}{2x^2 - 3x - 2}$

44. $\dfrac{3}{x^2 - 4x - 5} + \dfrac{1}{3x^2 + 8x + 5}$

45. $\dfrac{7}{3x^2 + 5x - 2} + \dfrac{3}{2x^2 + 11x + 14}$

46. $\dfrac{x - 1}{x^2 - 2x - 24} + \dfrac{1}{x^2 + 6x + 8}$

47. $\dfrac{x}{x^2 - 25} + \dfrac{1}{x^2 - 12x + 35}$

48. $\dfrac{x + 1}{x^2 - 10x + 24} + \dfrac{5}{x^2 - 6x + 8}$

49. $\dfrac{x + 4}{x^2 + 6x} + \dfrac{3}{x^2 + 3x - 18}$

50. $\dfrac{x + 5}{x^2 + 10x + 24} + \dfrac{4}{x^2 - 16}$

51. $\dfrac{x + 4}{x^2 + 5x + 6} + \dfrac{4}{x^2 + 2x}$

52. $\dfrac{x + 2}{x^2 + 7x + 12} + \dfrac{-6}{x^2 - 9}$

53. $\dfrac{x}{x^2 - 25} + \dfrac{-1}{x^2 - 8x + 15}$

54. $\dfrac{x + 1}{x^2 + 2x - 15} + \dfrac{x + 6}{x^2 + 8x + 15}$

55. $\dfrac{x - 1}{x^2 + 8x + 15} + \dfrac{x + 5}{x^2 + 7x + 12}$

56. $\dfrac{x - 6}{x^2 - 5x} + \dfrac{x - 1}{3x^2 - 10x - 25}$

57. $\dfrac{2x - 3}{x^2 - 4} + \dfrac{x - 3}{2x^2 - 4x}$

58. $\dfrac{x - 5}{3x^2 - 13x + 4} + \dfrac{x - 3}{2x^2 - 5x - 12}$

59. $\dfrac{x - 2}{2x^2 + 3x} + \dfrac{3x + 1}{2x^2 + 9x + 9}$

60. $\dfrac{x - 3}{6x^2 - 29x + 35} + \dfrac{3x - 1}{2x^2 + 3x - 20}$

These exercises involve differences of polynomial fractions. Recall that a difference of polynomial fractions can be changed into a sum of polynomial fractions by CHANGING THE SIGN OF EVERY TERM in the numerator of the fraction which is being subtracted.

Worked example:

$$\begin{aligned}
\frac{x}{x+3} - \frac{2x-5}{3x-4} &= \frac{x}{x+3} + \frac{-2x+5}{3x-4} \\
&= \frac{x(3x-4)}{(x+3)(3x-4)} + \frac{(-2x+5)(x+3)}{(3x-4)(x+3)} \\
&= \frac{3x^2-4x}{(x+3)(3x-4)} + \frac{-2x^2-x+15}{(3x-4)(x+3)} \\
&= \frac{(3x^2-4x) + (-2x^2-x+15)}{(x+3)(3x-4)} \\
&= \frac{x^2-5x+15}{(x+3)(3x-4)}
\end{aligned}$$

****

61. $\dfrac{x}{x+7} - \dfrac{2x+3}{x^2+3x-28}$

62. $\dfrac{x-4}{x^2-4x+3} - \dfrac{x+5}{x-3}$

63. $\dfrac{3x+5}{2x^2-5x-12} - \dfrac{x-2}{2x+3}$

64. $\dfrac{x + 6x - 4}{2x^2 - 5x} - \dfrac{x-7}{2x-5}$

65. $\dfrac{2x+3}{x+4} - \dfrac{5x-6}{3x^2+14x+8}$

66. $\dfrac{x+1}{x} - \dfrac{x-4}{x+3}$

67. $\dfrac{2x+1}{x-5} - \dfrac{x}{x-3}$

68. $\dfrac{x-2}{x-6} - \dfrac{x+4}{2x+3}$

69. $\dfrac{2x+3}{x+5} - \dfrac{3x-1}{2x-7}$

70. $\dfrac{x-5}{3x-2} - \dfrac{2x+7}{2x+5}$

The following exercises are subtraction problems: Remember to change the sign of each term in the numerator of the fraction which is being subtracted, then add. The resulting fraction can then be reduced.

<u>Worked example:</u>

$$\frac{2x + 1}{x^2 - 2x - 8} - \frac{x + 2}{x^2 - 4x} = \frac{2x + 1}{x^2 - 2x - 8} + \frac{-x - 2}{x^2 - 4x}$$

$$= \frac{2x + 1}{(x - 4)(x + 2)} + \frac{-x - 2}{x(x - 4)}$$

$$= \frac{(2x + 1)x}{(x - 4)(x + 2)x} + \frac{(-x - 2)(x + 2)}{x(x - 4)(x + 2)}$$

$$= \frac{(2x^2 + x) + (-x^2 - 4x - 4)}{x(x - 4)(x + 2)}$$

$$= \frac{x^2 - 3x - 4}{x(x - 4)(x + 2)}$$

$$= \frac{(x - 4)(x + 1)}{x(x - 4)(x + 2)}$$

$$= \frac{x + 1}{x(x + 2)}$$

****

71. $\dfrac{x}{x^2 + x - 6} - \dfrac{1}{x - 2}$

72. $\dfrac{x + 4}{x^2 + 4x + 3} - \dfrac{3}{x^2 - 9}$

73. $\dfrac{x + 6}{x^2 - 4} - \dfrac{1}{x^2 + 3x + 2}$

74. $\dfrac{x + 1}{x^2 - 7x + 10} - \dfrac{x}{x^2 - 6x + 8}$

75. $\dfrac{x + 1}{x^2 - x - 20} - \dfrac{x + 2}{x^2 + 2x - 8}$

The following problems involve the sums and/or differences of three polynomial fractions. The LCD is a product of three first degree polynomials. The resulting polynomial fraction can be reduced to lowest terms.

76. $\dfrac{x + 2}{x^2 + x} + \dfrac{3x}{x^2 - x - 2} + \dfrac{x - 1}{x^2 - 2x}$

77. $\dfrac{2x + 1}{x^2 + x - 12} + \dfrac{x - 1}{x^2 - 8x + 15} - \dfrac{x}{x^2 - x - 20}$

78. $\dfrac{x}{x^2 - 1} + \dfrac{x - 4}{x^2 + 4x - 5} - \dfrac{x + 3}{x^2 + 6x + 5}$

79. $\dfrac{2x + 3}{x^2 + 3x - 10} + \dfrac{x - 3}{x^2 - 3x + 2} + \dfrac{x}{x^2 - 6x + 5}$

80. $\dfrac{2x + 1}{x^2 - 4x - 21} - \dfrac{x - 4}{x^2 - 12x + 35} + \dfrac{x + 1}{x^2 - 2x - 15}$

81. $\dfrac{3x + 1}{2x^2 - x - 6} - \dfrac{2x - 5}{6x^2 + 5x - 6} + \dfrac{x - 6}{3x^2 - 8x + 4}$

82. $\dfrac{x - 2}{x^2 - x} + \dfrac{3x + 1}{2x^2 - 5x} - \dfrac{x + 2}{2x^2 - 7x + 5}$

83. $\dfrac{x + 4}{x^2 - 9} + \dfrac{3x - 2}{2x^2 - 5x - 3} + \dfrac{x - 2}{2x^2 + 7x + 3}$

84. $\dfrac{x + 3}{2x^2 - 3x} + \dfrac{2x + 5}{2x^2 + 5x - 12} - \dfrac{x - 4}{x^2 + 4x}$

85. $\dfrac{x - 4}{x^2 - 2x - 3} + \dfrac{x - 1}{x^2 + 2x - 15} - \dfrac{x - 2}{x^2 + 6x + 5}$

In this exercise set, the denominators have factors that are additive inverses of one another, for example $x - 5$ and $5 - x$. The LCD does not include both of these factors; a(−1) can be factored out of one of these factors to get similar factors in each denominator. There are two methods for adding the resulting fractions.

Method I: Leave the (−1) as a factor in the denominator. The LCD will have a (−1) as one of its factors.

Worked example:

$$\frac{2x+1}{x^2+2x-15} + \frac{4}{3-x} = \frac{2x+1}{(x-3)(x+5)} + \frac{4}{(-1)(x-3)}$$

$\boxed{\text{LCD} = (-1)(x-3)(x+5)}$

$$= \frac{(-1)(2x+1)}{(-1)(x-3)(x+5)} + \frac{4(x+5)}{(-1)(x-3)(x+5)}$$

$$= \frac{(-2x-1)+(4x+20)}{(-1)(x-3)(x+5)}$$

$$= \frac{2x+19}{(-1)(x-3)(x+5)}$$

Method II: Build up the fraction containing a (−1) as factor in the denominator by multiplying numerator and denominator by (−1).

Worked example:

$$\frac{x-2}{3x-x^2} + \frac{x+4}{x^2-2x-3} = \frac{x-2}{(-1)x(x-3)} + \frac{x+4}{(x+1)(x-3)}$$

$$= \frac{(-1)(x-2)}{(-1)(-1)x(x-3)} + \frac{x+4}{(x+1)(x-3)}$$

$$= \frac{-x+2}{x(x-3)} + \frac{x+4}{(x+1)(x-3)}$$

$$= \frac{(-x+2)(x+1)}{x(x-3)(x+1)} + \frac{(x+4)x}{(x+1)(x-3)x}$$

$$= \frac{(-x^2+x+2)+(x^2+4x)}{x(x+1)(x-3)}$$

$$= \frac{5x+2}{x(x+1)(x-3)}$$

Note that either method is acceptable. The answers in the back were obtained by using method II.

86. $\dfrac{2x + 3}{4 - x^2} + \dfrac{x}{x - 2}$

87. $\dfrac{x + 4}{x^2 - 3x - 10} + \dfrac{3x - 2}{5x - x^2}$

88. $\dfrac{x + 1}{2x^2 - x - 3} + \dfrac{x - 5}{3 - 2x}$

89. $\dfrac{2x - 1}{9 - x^2} + \dfrac{3x - 4}{x^2 - x - 6}$

90. $\dfrac{4x - 5}{3x - 2x^2} + \dfrac{7x + 2}{2x^2 + 3x - 9}$

This final exercise set is a collection of problems similar to those in all the previous exercises. All answers should be in lowest terms.

91. $\dfrac{x + 4}{2x - 3} + \dfrac{3x - 10}{2x - 3}$

92. $\dfrac{4x + 7}{3x^2} + \dfrac{3x - 2}{15x}$

93. $\dfrac{3x - 4}{12x - 18} + \dfrac{2x + 5}{18x - 27}$

94. $\dfrac{2x - 5}{x^2 + 4x - 21} + \dfrac{x + 2}{x - 3}$

95. $\dfrac{3x + 2}{2x + 5} + \dfrac{3x + 11}{8x^2 + 14x - 15}$

96. $\dfrac{5x - 6}{12x^2 - 7x - 45} + \dfrac{3x + 7}{4x - 9} + \dfrac{5 - 2x}{3x + 5}$

97. $\dfrac{12}{4x^2 + 8x - 5} + \dfrac{-5}{2x^2 + 5x}$

98. $\dfrac{x - 6}{x^2 - 4} + \dfrac{7}{x^2 + 3x - 10}$

99. $\dfrac{2x + 5}{x^2 + 5x - 6} + \dfrac{x + 3}{x^2 - 6x + 5}$

100. $\dfrac{x}{6x^2 - 7x - 3} + \dfrac{x - 3}{10x^2 - 19x - 6}$

101. $\dfrac{3x - 5}{2x + 1} - \dfrac{4x + 7}{6x^2 - 5x - 4}$

102. $\dfrac{x + 7}{x^2 - 2x - 3} - \dfrac{x + 2}{x^2 - 4x + 3}$

103. $\dfrac{5x + 3}{x^2 - 5x - 6} - \dfrac{3x - 2}{x^2 - 4x - 12} - \dfrac{x + 3}{x^2 + 3x + 2}$

104. $\dfrac{x + 4}{6x^2 - 7x - 3} - \dfrac{2x - 1}{3x^2 + 7x + 2} + \dfrac{x}{2x^2 + x - 6}$

105. $\dfrac{2x + 7}{x^2 - 2x - 35} + \dfrac{x + 3}{28 + 3x - x^2}$

2. $\frac{3x - 3}{x - 4}$

4. $\frac{5x - 4}{3x + 1}$

6. $\frac{x + 6x - 3}{x^2}$

8. $\frac{5x + 19x - 6}{10x^2}$

10. $\frac{2x - 7y}{5x^2 y}$

12. $\frac{9x - 8}{10(x + 3)}$

14. $\frac{6x - 13}{4(3x + 1)}$

16. $\frac{x^2 + 2x + 3}{(x + 2)(x - 2)}$

18. $\frac{x^2 + x + 2}{(x + 1)(x - 5)}$

20. $\frac{x^2 - 4x + 1}{(x + 2)(x - 5}$

22. $\frac{2x^2 + 4x - 7}{(3x - 7)(x - 5)}$

24. $\frac{2x^2 + 8x - 4}{(2x + 5)(3x - 4)}$

26. $\frac{x^2 - 4x + 15}{(x + 5)(x - 7)}$

28. $\frac{x^2 + 4x - 11}{(3x + 1)(x - 3)}$

30. $\frac{x^2 + x + 38}{(x - 6)(3x + 2)}$

32. $\frac{7x^2 + 1}{(3x - 4)(2x + 1)}$

34. $\frac{9x^2 + 21x - 1}{(3x + 1)(3x - 2)}$

36. $\frac{2x^2 + 11x + 25}{(x + 8)(x - 3)}$

38. $\frac{3x^2 + 9x + 15}{(x + 4)(3x - 5)}$

40. $\frac{8x^2 + 34}{(3x - 7)(2x + 5)}$

42. $\frac{7}{(x + 3)(x - 4)}$

44. $\frac{10}{(x - 5)(3x + 5)}$

46. $\frac{x - 2}{(x - 6)(x + 2)}$

48. $\frac{x + 8}{(x - 6)(x - 2)}$

50. $\frac{x + 1}{(x + 6)(x - 4)}$

52. $\frac{x - 10}{(x + 4)(x - 3)}$

54. $\frac{2x - 3}{(x - 3)(x + 3)}$

56. $\frac{2(2x + 3)}{x(3x + 5)}$

58. $\frac{5x + 3}{(3x - 1)(2x + 3)}$

60. $\frac{5x + 1}{(3x - 7)(x + 4)}$

62. $\frac{-x - 3x + 1}{(x - 3)(x - 1)}$

64. $\frac{13x - 4}{x(2x - 5)}$

66. $\frac{8x + 3}{x(x + 3)}$

68. $\frac{x^2 + x + 18}{(x - 6)(2x + 3)}$

70. $\frac{-4x^2 - 22x - 11}{(3x - 2)(2x + 5)}$

72. $\frac{x - 5}{(x + 1)(x - 3)}$

74. $\frac{2}{(x - 5)(x - 4)}$

76. $\frac{5(x - 1)}{x(x - 2)}$

78. $\frac{1}{x + 5}$

80. $\frac{2x}{(x + 3)(x - 5)}$

82. $\frac{4x - 9}{x(2x - 5)}$

84. $\frac{x + 23}{(x + 4)(2x - 3)}$

86. $\frac{x^2 - 3}{(x + 2)(x - 2)}$

88. $\frac{-x^2 + 5x + 6}{(2x - 3)(x + 1)}$

90. $\frac{3x^2 - 5x + 15}{x(2x - 3)(x + 3)}$

92. $\frac{3x^2 + 18x + 35}{15x^2}$

94. $\frac{x^2 + 11x + 9}{(x + 7)(x - 3)}$

96. $\frac{x^2 + 79x - 16}{(4x - 9)(3x + 5)}$

98. $\frac{x + 8}{(x + 2)(x + 5)}$

100. $\frac{4x + 1}{(3x + 1)(5x - 2)}$

102. $\frac{3}{(x + 1)(x - 1)}$

104. $\frac{5}{(2x - 3)(x + 2)}$

1. $\frac{x + 4}{x}$ 3. $\frac{4x + 5}{2x + 5}$ 5. $\frac{x + 3x - 3}{x^2 + 3x}$ 7. $\frac{3xy + 15y - 5}{6xy}$

9. $\frac{6x - 13xy + 6y}{12x^2 y}$ 11. $\frac{2x + 5x - 17}{8(x - 2)}$ 13. $\frac{2x + 3x - 2}{6(2x + 5)}$

15. $\frac{10x - 5}{12(2x - 5)}$ 17. $\frac{x^2 - x + 7}{(x + 3)(x - 2)}$ 19. $\frac{x^2 + 5x + 1}{(x + 3)}$

21. $\frac{2x^2 + 10x + 10}{(2x + 3)(x - 5)}$ 23. $\frac{32x + 12}{(6x + 1)(x - 3)}$ 25. $\frac{4x^2 + 3x - 14}{(2x + 3)(5x - 7)}$

27. $\frac{11x - 6}{(x + 2)(2x - 3)}$ 29. $\frac{5x^2 + 24x + 25}{(x + 3)(2x + 7)}$ 31. $\frac{2x^2 + 9x + 21}{(2x + 3)(2x - 5)}$

33. $\frac{6x^2 - x + 15}{(2x - 7)(4x + 3)}$ 35. $\frac{2x^2 + 13x - 13}{(6x + 1)(2x - 5)}$ 37. $\frac{5x^2 + 3x - 15}{(2x + 3)(x - 6)}$

39. $\frac{8x^2 + 3x + 12}{(2x + 5)(2x - 5)}$ 41. $\frac{3}{(x + 2)(x + 5)}$ 43. $\frac{9}{(x - 4)(2x + 1)}$

45. $\frac{23}{(3x - 1)(2x + 7)}$ 47. $\frac{x - 1}{(x + 5)(x - 7)}$ 49. $\frac{x - 2}{x(x - 3)}$

51. $\frac{x + 6}{x(x + 3)}$ 53. $\frac{x + 1}{(x + 5)(x - 3)}$ 55. $\frac{2x + 7}{(x + 5)(x + 4)}$

57. $\frac{5x + 3}{2x(x + 2)}$ 59. $\frac{2(x - 1)}{x(x + 3)}$ 61. $\frac{x^2 - 6x - 3}{(x + 7)(x - 4)}$

63. $\frac{-x^2 + 9x - 3}{(2x + 3)(x - 4)}$ 65. $\frac{6x^2 + 8x + 12}{(3x + 2)(x + 4)}$ 67. $\frac{x^2 - 3}{(x - 5)(x - 3)}$

69. $\frac{x^2 - 22x - 11}{(x + 5)(2x - 7)}$ 71. $\frac{-3}{(x + 3)(x - 2)}$ 73. $\frac{x + 4}{(x - 2)(x + 1)}$

75. $\frac{2}{(x - 2)(x - 5)}$ 77. $\frac{2x + 3}{(x + 4)(x - 5)}$ 79. $\frac{4x + 9}{(x - 1)(x - 5)}$

81. $\frac{3(3x + 5)}{(2x + 3)(3x - 2)}$ 83. $\frac{3x + 4}{(x + 3)(x - 3)}$ 85. $\frac{x + 9}{(x + 1)(x + 5)}$

87. $\frac{-2x^2 + 4}{x(x - 5)(x + 2)}$ 89. $\frac{x^2 + 2x - 10}{(x + 3)(x - 3)(x + 2)}$ 91. $\frac{2}{1}$

93. $\frac{13x - 2}{18(2x - 3)}$ 95. $\frac{12x^2 + 2x + 6}{(2x + 5)(4x - 3)}$ 97. $\frac{1}{(2x - 1)(x)}$

99. $\frac{3x + 7}{(x + 6)(x - 5)}$ 101. $\frac{9x^2 - 31x + 13}{(2x + 1)(3x - 4)}$ 103. $\frac{x + 13}{(x + 1)(x - 6)}$

105. $\frac{x^2 + 7x + 13}{(x + 5)(x - 7)(x + 4)}$