**IMPORTANT CONCEPTS**

A vector is a quantity described by both a magnitude and a direction.

The vector describes the situation at this point.

The length or magnitude is denoted \( A \). Magnitude is a scalar.

Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors \( \hat{i} \) and \( \hat{j} \) define the directions of the \( x \)- and \( y \)-axes.

**USING VECTORS**

**Components**

The component vectors are parallel to the \( x \)- and \( y \)-axes:

\[
\vec{A} = A_x \hat{i} + A_y \hat{j}
\]

In the figure at the right, for example:

\[
\begin{align*}
A_x &= A \cos \theta \\
A_y &= A \sin \theta \\
\theta &= \tan^{-1}(A_y/A_x)
\end{align*}
\]

- Minus signs need to be included if the vector points down or left.

**Working Graphically**

Addition

Negative

Subtraction

Multiplication

**Working Algebraically**

Vector calculations are done component by component: \( \vec{C} = 2\vec{A} + \vec{B} \) means

\[
\begin{align*}
C_x &= 2A_x + B_x \\
C_y &= 2A_y + B_y
\end{align*}
\]

The magnitude of \( \vec{C} \) is then \( C = \sqrt{C_x^2 + C_y^2} \) and its direction is found using \( \tan^{-1} \).
Questions and Example Problems from Chapter 3

Conceptual Question 3.8
Suppose two vectors have unequal magnitudes. Can their sum be zero? Explain.

3.8. No, it is not possible for two vectors with unequal magnitudes to add to zero. To add to zero, two vectors must be antiparallel and of the same length (magnitude).

Problem 3.4
A velocity vector 40° below the positive x-axis has a y-component of -10 m/s. What is the value of its x-component?

3.4. Visualize: The figure shows the components $v_x$ and $v_y$, and the angle $\theta$.

Solve: We have $v_y = -v \sin \theta$ where we have manually inserted the minus sign because $v_y$ points in the negative-y direction. The x-component is $v_x = v \cos \theta$. Taking the ratio $v_x/v_y$ and solving for $v_x$ gives $v_x = -v_y (\tan \theta)^{-1} = -(-10 \text{ m/s})(\tan 40^\circ)^{-1} = 12 \text{ m/s}$.

Assess: The x-component is positive since the position vector is in the fourth quadrant.

Problem 3.7
Draw each of the following vectors. Then find its x- and y-components.

a) $\vec{v} = (3.5 \text{ m/s, negative x-direction})$

b) $\vec{a} = (1.5 \text{ m/s}^2, 30^\circ \text{ above the negative x-axis})$

c) $\vec{F} = (50.0 \text{ N, } 36.9^\circ \text{ counterclockwise from the positive y-axis})$

3.7. Visualize:
Solve:  (a) \( v_x = (7.5 \text{ m/s})(\sin30^\circ) = 3.8 \text{ m/s}; \ v_y = (7.5 \text{ m/s})(\cos30^\circ) = 6.5 \text{ m/s} \)

(b) \( a_x = -(1.5 \text{ m/s}^2)(\cos30^\circ) = -1.3 \text{ m/s}^2; \ a_y = (1.5 \text{ m/s}^2)(\sin30^\circ) = 0.80 \text{ m/s}^2 \)

(c) \( F_x = -(50.0 \text{ N})(\sin36.9^\circ) = -30 \text{ N}; \ F_y = (50.0 \text{ N})(\cos36.9^\circ) = 40 \text{ N} \)

Problem 3.10
Draw each of the following vectors, label an angle that specifies the vector’s direction, then find its magnitude and direction.

a) \( \vec{B} = -4\hat{i} + 4\hat{j} \)

b) \( \vec{r} = (-2.0\hat{i} - 1.0\hat{j}) \text{ cm} \)

c) \( \vec{v} = (-10\hat{i} - 100\hat{j}) \text{ m/s} \)

d) \( \vec{a} = (20\hat{i} + 10\hat{j}) \text{ m/s}^2 \)

3.10. Visualize:

Solve:  (a) Using the formulas for the magnitude and direction of a vector, we have:
\[
B = \sqrt{(-4)^2 + (4)^2} = 5.7, \ \theta = \tan^{-1}\left(\frac{4}{4}\right) = 45^\circ
\]

(b) \( r = \sqrt{(-2.0 \text{ cm})^2 + (-1.0 \text{ cm})^2} = 2.2 \text{ cm}, \ \theta = \tan^{-1}\left(\frac{1.0}{2.0}\right) = 27^\circ \)

(c) \( v = \sqrt{(-10 \text{ m/s})^2 + (-100 \text{ m/s})^2} = 100 \text{ m/s}, \ \theta = \tan^{-1}\left(\frac{100}{10}\right) = 84^\circ \)

(d) \( a = \sqrt{(10 \text{ m/s}^2)^2 + (20 \text{ m/s}^2)^2} = 22 \text{ m/s}^2, \ \theta = \tan^{-1}\left(\frac{10}{20}\right) = 27^\circ \)
Problem 3.14
Let \( \vec{A} = 4\hat{i} - 2\hat{j}, \vec{B} = -3\hat{i} + 5\hat{j}, \) and \( \vec{D} = \vec{A} - \vec{B}. \)

a) Write vector \( \vec{D} \) in component form.
b) Draw a coordinate system and on it show vectors \( \vec{A}, \vec{B}, \) and \( \vec{D}. \)
c) What are the magnitude and direction of vector \( \vec{D} \)?

3.14. Visualize:

\[ \begin{align*}
\vec{A} &= 4\hat{i} - 2\hat{j} \\
\vec{B} &= -3\hat{i} + 5\hat{j}
\end{align*} \]

Solve: (a) We have \( \vec{A} = 4\hat{i} - 2\hat{j}, \vec{B} = -3\hat{i} + 5\hat{j}, \) and \( -\vec{B} = 3\hat{i} - 5\hat{j}. \) Thus, \( \vec{D} = \vec{A} + (-\vec{B}) = (4 + 3)\hat{i} + (-2 - 5)\hat{j} = 7\hat{i} - 7\hat{j}. \)

(b) Vectors \( \vec{A}, \vec{B}, \) and \( \vec{D} \) are shown in the figure above.

(c) Since \( \vec{D} = 7\hat{i} - 7\hat{j} = D_x\hat{i} + D_y\hat{j}, \) \( D_x = 7 \) and \( D_y = -7. \) Therefore, the magnitude and direction of \( \vec{D} \) are

\[ D = \sqrt{(7)^2 + (-7)^2} = 7\sqrt{2} = 9.9 \quad \theta = \tan^{-1}\left(\frac{|D_y|}{|D_x|}\right) = \tan^{-1}(7/7) = 45^\circ \]

Assess: Since \( |D_y| = |D_x|, \) the angle \( \theta = 45^\circ, \) as expected.

Problem 3.16
Let \( \vec{A} = 4\hat{i} - 2\hat{j}, \vec{B} = -3\hat{i} + 5\hat{j}, \) and \( \vec{F} = \vec{A} - 4\vec{B}. \)

a) Write vector \( \vec{F} \) in component form.
b) Draw a coordinate system and on it show vectors \( \vec{A}, \vec{B}, \) and \( \vec{F}. \)
c) What are the magnitude and direction of vector \( \vec{F} \)?

3.16. Visualize:
Solve: (a) We have \( \vec{A} = 4\hat{i} - 2\hat{j} \) and \( \vec{B} = -3\hat{i} + 5\hat{j} \). This means \( 4\vec{B} = -12\hat{i} + 20\hat{j} \). Hence, \( \vec{F} = \vec{A} - 4\vec{B} = (4 - (-12))\hat{i} + (-2 - 20)\hat{j} = 16\hat{i} - 22\hat{j} = F_x\hat{i} + F_y\hat{j} \), so \( F_x = 16 \) and \( F_y = -22 \).

(b) The vectors \( \vec{A}, \vec{B}, \) and \( \vec{F} \) are shown in the above figure.

(c) The magnitude and direction of \( \vec{F} \) are
\[
F = \sqrt{F_x^2 + F_y^2} = \sqrt{(16)^2 + (-22)^2} = 27
\]
\[
\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-22}{16}\right) = 54^\circ
\]

Assess: \( F_y > F_x \) implies \( \theta > 45^\circ \), which is consistent with the figure.

Problem 3.26
The figure below shows vectors \( \vec{A} \) and \( \vec{B} \). Find \( \vec{D} = 2\vec{A} + \vec{B} \). Write your answer in component form.

3.26. Visualize:
Solve: In the tilted coordinate system, the vectors \( \vec{A} \) and \( \vec{B} \) are expressed as:
\[
\vec{A} = [2\sin(15^\circ) \text{ m}]\hat{i} + [2\cos(15^\circ) \text{ m}]\hat{j} \quad \text{and} \quad \vec{B} = [4\cos(15^\circ) \text{ m}]\hat{i} - [4\sin(15^\circ) \text{ m}]\hat{j}.
\]
Therefore, \( \vec{D} = 2\vec{A} + \vec{B} = (4 \text{ m})[\sin(15^\circ) + \cos(15^\circ)]\hat{i} + (4 \text{ m})[\cos(15^\circ) - \sin(15^\circ)]\hat{j} = (4.9 \text{ m})\hat{i} + (2.9 \text{ m})\hat{j} \).

Assess: The magnitude of this vector is \( D = \sqrt{(4.9 \text{ m})^2 + (2.9 \text{ m})^2} = 5.7 \text{ m} \), and it makes an angle of \( \theta = \tan^{-1}(2.9 \text{ m}/4.9 \text{ m}) = 31^\circ \) with the +x-axis. The resultant vector can be obtained graphically by using the rule of tail-to-tip addition.

Problem 3.44
Four forces are exerted on the object shown in the figure below. (Forces are measured in Newtons, abbreviated N.) The net force on the object is \( \vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = 4.0\hat{i}N \). What are (a) \( \vec{F}_3 \) and (b) \( \vec{F}_4 \)? Give your answers in component form.

3.44. Visualize: \( \vec{F}_3 \) and \( \vec{F}_4 \) are along the axes. The vertical components must add to zero. We treat the horizontal and vertical components separately.

Solve: First find the component vectors of \( \vec{F}_2 \).
\[
\vec{F}_1 = (0.0 \text{ N})\hat{i} - (5.0 \text{ N})\hat{j} \quad \vec{F}_2 = (6.0 \text{ N})\cos20^\circ\hat{i} + (6.0 \text{ N})\sin20^\circ\hat{j} = (5.6 \text{ N})\hat{i} + (2.1 \text{ N})\hat{j}
\]
There is no \( \hat{i} \) component of \( \vec{F}_3 \).
\[
\vec{F}_3 + (\vec{F}_1) + (\vec{F}_2) = (0.0 \text{ N})\hat{i} + (5.6 \text{ N})\hat{j} - (2.1 \text{ N})\hat{j} = (2.9 \text{ N})\hat{j}
\]
There is no \( \hat{j} \) component of \( \vec{F}_4 \).
\[
\vec{F}_4 + (\vec{F}_2) = (4.0 \text{ N})\hat{i} + (5.6 \text{ N})\hat{i} - (1.6 \text{ N})\hat{i} = (-1.6 \text{ N})\hat{i}
\]
Assess: The figure could have been drawn differently to give negative values for both components; the magnitudes would be the same, however.

Problem A
For the vectors \( \vec{a} = (3.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j} \) and \( \vec{b} = (5.0 \text{ m})\hat{i} + (-2.0 \text{ m})\hat{j} \), give \( \vec{a} + \vec{b} \) in (a) unit-vector notation, and as (b) a magnitude and (c) an angle (relative to \( \hat{i} \)). Now give \( \vec{b} - \vec{a} \) in (d) unit-vector notation, and as (e) magnitude and (f) an angle.
Problem B

Find the sum of the following four vectors in (a) unit-vector notation, and as (b) a magnitude and (c) an angle relative to +x.

\( \vec{P} \): 10.0 m, at 25.0° counterclockwise from +x
\( \vec{Q} \): 12.0 m, at 10.0° counterclockwise from +y
\( \vec{R} \): 8.00 m, at 20.0° clockwise from -y
\( \vec{S} \): 9.00 m, at 40.0° counterclockwise from -y

\( \vec{A}_x = \vec{P}_x + \vec{Q}_x + \vec{R}_x + \vec{S}_x = (9.06 \text{ m}) \cos 25.0^\circ \hat{\imath} + (10.0 \text{ m}) \cos 10.0^\circ \hat{\jmath} = (9.06 \text{ m}) \cos 25.0^\circ \hat{\imath} + (10.0 \text{ m}) \cos 10.0^\circ \hat{\jmath} = 10.0 \text{ m} \hat{\imath} \\
\vec{A}_y = \vec{P}_y + \vec{Q}_y + \vec{R}_y + \vec{S}_y = (9.06 \text{ m}) \cos 25.0^\circ \hat{\imath} + (10.0 \text{ m}) \cos 10.0^\circ \hat{\jmath} = (9.06 \text{ m}) \cos 25.0^\circ \hat{\imath} + (10.0 \text{ m}) \cos 10.0^\circ \hat{\jmath} = 10.0 \text{ m} \hat{\jmath} \)

\( a) \ \vec{A} = (10.0 \text{ m}) \hat{\imath} + (1.63 \text{ m}) \hat{\jmath} \)

\( b) \ A = \sqrt{A_x^2 + A_y^2} = \sqrt{(10.0 \text{ m})^2 + (1.63 \text{ m})^2} = 10.2 \text{ m} \)

\( c) \ \theta = \tan^{-1} \left( \frac{A_y}{A_x} \right) = \tan^{-1} \left( \frac{1.63 \text{ m}}{10.0 \text{ m}} \right) \rightarrow \theta = 9.35^\circ \)