Chapter 6 Lecture

In this chapter, you will learn to solve linear force-and-motion problems.

How are Newton's laws used to solve problems?
- Newton's first and second laws are vector equations. To use them,
  - Draw a free-body diagram,
  - Read the x- and y-components of the forces directly off the free-body diagram,
  - Use $\sum F_x = ma_x$ and $\sum F_y = ma_y$.

Chapter 6 Preview

How are dynamics problems solved?
- A net force on an object causes the object to accelerate,
  - Identify the forces and draw a free-body diagram,
  - Use Newton's second law to find the object's acceleration,
  - Use kinematics for velocity and position.

Looking Back: Sections 2.4-2.6: Kinematics
Chapter 6 Preview

How are equilibrium problems solved?
An object at rest or moving with constant velocity is in equilibrium with no net force.
- Identify the forces and draw a free-body diagram.
- Use Newton’s second law with \( \sum \vec{F} = 0 \) to solve for unknown forces.
  - LOOKING BACK Sections 5.1–5.2 Forces

Chapter 6 Preview

What are mass and weight?
Mass and weight are not the same.
- Mass describes an object’s inertia. Loosely speaking, it is the amount of matter in an object. It is the same everywhere.
- Gravity is a force.
- Weight is the result of weighing an object on a scale. It depends on mass, gravity, and acceleration.

Chapter 6 Preview

How do we model friction and drag?
Friction and drag are complex forces, but we will develop simple models of each.
- Static, kinetic, and rolling friction depend on the coefficients of friction but not on the object’s speed.
- Drag depends on the square of an object’s speed and on its cross-section area.
- Falling objects reach terminal speed when drag and gravity are balanced.

Chapter 6 Preview

How do we solve problems?
We will develop and use a four-part problem-solving strategy:
- Model the problem, using information about objects and forces.
- Visualize the situation with a pictorial representation.
- Set up and solve the problem with Newton’s laws.
- Assess the result to see if it is reasonable.
In the absence of a net force, an object is at rest or moves with constant velocity.

- Its acceleration is zero, and we say that it is in equilibrium.
- The concept of equilibrium is essential for the engineering analysis of stationary objects such as bridges.
- When the acceleration is zero, then Newton’s second law in two dimensions becomes:
  \[(F_{net})_x = \sum (F)_x = 0 \quad \text{and} \quad (F_{net})_y = \sum (F)_y = 0\]

Example 6.2 Towing a Car up a Hill

A car with a weight of 15,000 N is being towed up a 20° slope at constant velocity. Friction is negligible. The tow rope is rated at 6000 N maximum tension. Will it break?

MODEL Model the car as a particle in equilibrium.

Mechanical Equilibrium

The Equilibrium Model

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- When the acceleration is zero, then Newton’s second law in two dimensions becomes:
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The essence of Newtonian mechanics can be expressed in two steps:

- The forces on an object determine its acceleration $\ddot{\mathbf{a}} = \mathbf{F}_{\text{net}} / m$.
- The object’s trajectory can be determined by using $\ddot{\mathbf{a}}$ in the equations of kinematics.
Problem-Solving Strategy: Newtonian Mechanics

### Problem-Solving Strategy 6.1

**Newtonian mechanics**
- **Model:** Model the object as a particle. Make other simplifications depending on what kinds of forces are acting.
- **Visualize:** Draw a pictorial representation.
  - Show important points in the motion with a sketch, establish a coordinate system, define symbols, and identify what the problem is trying to find.
  - Use a motion diagram to determine the object’s acceleration vector. The acceleration is zero for an object in equilibrium.
  - Identify all forces acting on the object at this instant and show them on a free-body diagram.
- **If it’s OK to go back and forth between these steps as you visualize the situation.**

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**Example 6.3 Speed of a Towed Car**

**EXAMPLE 6.3** Speed of a towed car

A 1500 kg car is pulled by a tow truck. The tension in the tow rope is 2500 N, and a 200 N friction force opposes the motion. If the car starts from rest, what is its speed after 5.0 seconds?

**Model:** Model the car as an accelerating particle. We’ll assume, as part of our interpretation of the problem, that the road is horizontal and that the direction of motion is to the right.

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**Example 6.3 Speed of a Towed Car**

**EXAMPLE 6.3** Speed of a towed car

**Visualize:** Figure 6.3 shows the pictorial representation. We’ve established a coordinate system and defined symbols to represent kinematic quantities. We’ve identified the speed $v_x$, rather than the velocity $v_x$, as what we’re trying to find.
Example 6.3 Speed of a Towed Car

**Example 6.3 Speed of a Towed Car**

**Statement:**
When the force vectors are additive given $F_{\text{net}}$, we can find the net force on the system. The vectors are path-independent, with $F_{\text{net}}$ given by:

\[
F_{\text{net}} = \sum F_x = F_x + F_y = 0
\]

**Details:**
- $F_x = F - F_g - F_{\text{f}}$
- $F_y = 0$

**Solution:**
- $F_{\text{net}} = 0$
- $F = F_{\text{f}}$
- $F_{\text{net}} = F_{\text{f}}$

**Diagram:**
- A diagram showing the forces acting on the car, including tension $T$, friction $F_{\text{f}}$, and normal force $N$.

**Example 6.3 Speed of a Towed Car**

**Statement:**
For the car, which has a mass $m$, acting on the system, the net force is given by:

\[
F_{\text{net}} = m\alpha = \frac{v^2}{r}
\]

**Details:**
- $m\alpha = \frac{v^2}{r}$
- $\alpha = \frac{v^2}{mr}$

**Solution:**
- $v = \sqrt{\frac{mr}{m}}$
- $v = \sqrt{\frac{mr}{m}}$

**Diagram:**
- A diagram showing the forces acting on the car, including tension $T$, friction $F_{\text{f}}$, and normal force $N$.
Mass: An Intrinsic Property

- A pan balance, shown in the figure, is a device for measuring mass.
- The measurement does not depend on the strength of gravity.
- Mass is a scalar quantity that describes an object’s inertia.
- Mass describes the amount of matter in an object.
- Mass is an intrinsic property of an object.

Gravity: A Force

- Gravity is an attractive, long-range force between any two objects.
- The figure shows two objects with masses $m_1$ and $m_2$ whose centers are separated by distance $r$.
- Each object pulls on the other with a force:
  \[ F_{1 \text{ on } 2} = F_{2 \text{ on } 1} = \frac{GM_1 m_2}{r^2} \]  (Newton’s law of gravity)
  where $G = 6.67 \times 10^{-11}$ N m$^2$/kg$^2$ is the gravitational constant.

Gravity: A Force

- The gravitational force between two human-sized objects is very small.
- Only when one of the objects is planet-sized or larger does gravity become an important force.
- For objects near the surface of the planet earth,
  \[ F_G = F_{\text{down on } m} = \frac{GMm}{R^2} \times \text{ straight down} \quad \text{and} \quad mg \times \text{ straight down} \]
  where $M$ and $R$ are the mass and radius of the earth, and $g = 9.80$ m/s$^2$. 

Gravity: A Force

- The magnitude of the gravitational force is \( F_G = mg \), where \( g = \frac{GM}{R^2} \).
- The figure shows the free-body diagram of an object in free fall near the surface of a planet.
- With \( \vec{F}_{\text{net}} = \vec{F}_G \) Newton's second law predicts the acceleration to be
  \[ a_{\text{free-fall}} = \frac{F_{\text{net}}}{m} = \frac{F_G}{m} = g \text{ (g, straight down)} \]
- All objects on the same planet, regardless of mass, have the same free-fall acceleration!

Weight: A Measurement

- When you weigh yourself, you stand on a spring scale and compress a spring.
- The reading of a spring scale is \( F_{\text{sp}} \), the magnitude of the upward force the spring is exerting.
- Let's define the weight of an object to be the reading \( F_{\text{sp}} \) of a calibrated spring scale when the object is at rest relative to the scale.
- That is, weight is a measurement, the result of "weighing" an object.
- Because \( F_{\text{sp}} \) is a force, weight is measured in newtons.

Weight: A Measurement

- A bathroom scale uses compressed springs which push up.
- When any spring scale measures an object at rest, \( \vec{F}_{\text{net}} = \vec{0} \).
- The upward spring force exactly balances the downward gravitational force of magnitude \( mg \):
  \[ F_{\text{sp}} = F_G = mg \]
- Weight is defined as the magnitude of \( F_{\text{sp}} \) when the object is at rest relative to the stationary scale:
  \[ w = mg \text{ (weight of a stationary object)} \]
- The figure shows a man weighing himself in an accelerating elevator.
  Looking at the free-body diagram, the \( y \)-component of Newton's second law is:
  \[ (F_{\text{net},y}) = (F_{\text{sp},y}) + (F_{G,y}) = F_{\text{sp}} - mg = ma_y \]
  \[ w = \text{scale reading } F_{\text{sp}} = mg + ma_y = mg \left( 1 + \frac{a_y}{g} \right) \]
- You weigh more as an elevator accelerates upward!
**Weightlessness**

- The weight of an object which accelerates vertically is
  \[ w = \text{scale reading} \times mg = ma_y = mg \left( 1 + \frac{a_y}{g} \right) \]
- If an object is accelerating downward with \( a_y = -g \), then \( w = 0 \).
- An object in free fall has no weight!
- Astronauts while orbiting the earth are also weightless.
- Does this mean that they are in free fall?

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**Static Friction**

- A shoe pushes on a wooden floor but does not slip.
- On a microscopic scale, both surfaces are “rough” and high features on the two surfaces form molecular bonds.
- These bonds can produce a force tangent to the surface, called the static friction force.
- Static friction is a result of many molecular springs being compressed or stretched ever so slightly.

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**Static Friction**

- The figure shows a rope pulling on a box that, due to static friction, isn’t moving.
- Looking at the free-body diagram, the \( x \)-component of Newton’s first law requires that the static friction force must exactly balance the tension force:
  \[ f_x = T \]
- \( f_x \) points in the direction opposite to the way the object would move if there were no static friction.

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**Static Friction**

- Static friction acts in response to an applied force.
  \[ \vec{f}_s \]
  \( \vec{F} \) is balanced by \( \vec{f}_s \) and the box does not move.
- As \( T \) increases, \( f_x \) grows...
  \[ f_x \] \( f_{x \text{ max}} \)
  \( \vec{F} \)
  
  ... until \( f_x \) reaches \( f_{x \text{ max}} \). Now, if \( T \) gets any bigger, the object will start to move.
Static Friction

- Static friction force has a maximum possible size $f_{s_{\text{max}}}$. 
- An object remains at rest as long as $f_s < f_{s_{\text{max}}}$.
- The object just begins to slip when $f_s = f_{s_{\text{max}}}$.
- A static friction force $f_s > f_{s_{\text{max}}}$ is not physically possible:

$$f_{s_{\text{max}}} = \mu_s n$$

where the proportionality constant $\mu_s$ is called the coefficient of static friction.

Kinetic Friction

- The kinetic friction force is proportional to the magnitude of the normal force:

$$f_k = \mu_k n$$

where the proportionality constant $\mu_k$ is called the coefficient of kinetic friction.
- The kinetic friction direction is opposite to the velocity of the object relative to the surface.
- For any particular pair of surfaces, $\mu_k < \mu_s$.

Rolling Motion

- If you slam on the brakes hard enough, your car tires slide against the road surface and leave skid marks. This is kinetic friction.
- A wheel rolling on a surface also experiences friction, but not kinetic friction.
- The portion of the wheel that contacts the surface is stationary with respect to the surface, not sliding.
- The interaction of this contact area with the surface causes rolling friction.

Rolling Friction

- A car with no engine or brakes applied does not roll forever; it gradually slows down.
- This is due to rolling friction.
- The force of rolling friction can be calculated as

$$f_r = \mu_r n$$

where $\mu_r$ is called the coefficient of rolling friction.
- The rolling friction direction is opposite to the velocity of the rolling object relative to the surface.
Coefficients of Friction

<table>
<thead>
<tr>
<th>Materials</th>
<th>Static $\mu_s$</th>
<th>Kinetic $\mu_k$</th>
<th>Rolling $\mu_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber on dry concrete</td>
<td>1.00</td>
<td>0.80</td>
<td>0.02</td>
</tr>
<tr>
<td>Rubber on wet concrete</td>
<td>0.30</td>
<td>0.25</td>
<td>0.02</td>
</tr>
<tr>
<td>Steel on steel (dry)</td>
<td>0.80</td>
<td>0.60</td>
<td>0.002</td>
</tr>
<tr>
<td>Steel on steel (lubricated)</td>
<td>0.10</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.50</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td>Wood on snow</td>
<td>0.12</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>Ice on ice</td>
<td>0.10</td>
<td>0.03</td>
<td></td>
</tr>
</tbody>
</table>

A Model of Friction

**Friction**
- The friction force is parallel to the surface.
  - Static friction: Acts as needed to prevent motion. Can have any magnitude up to $F_{\text{max}} = \mu_s F_n$.
  - Kinetic friction: Opposes motion with $F = \mu_k F_n$.
  - Rolling friction: Opposes motion with $F = \mu_r F_n$.
- Graphically:

Causes of Friction

- All surfaces are very rough on a microscopic scale.
- When two surfaces are pressed together, the high points on each side come into contact and form molecular bonds.
- The amount of contact depends on the normal force $F_n$.
- When the two surfaces are sliding against each other, the bonds don’t form fully, but they do tend to slow the motion.

Drag

- The air exerts a drag force on objects as they move through the air.
- Faster objects experience a greater drag force than slower objects.
- The drag force on a high-speed motorcyclist is significant.
- The drag force direction is opposite the object’s velocity.
Drag

- For normal-sized objects on earth traveling at a speed $v$ which is less than a few hundred meters per second, air resistance can be modeled as
  \[ F_{\text{drag}} = \frac{1}{2} \rho \pi A v^2 \text{ (direction opposite the motion)} \]
- $A$ is the cross-section area of the object.
- $\rho$ is the density of the air, which is $1.3 \text{ kg/m}^3$, at atmospheric pressure and $0^\circ \text{C}$, a common reference point of pressure and temperature.
- $C$ is the drag coefficient, which is a dimensionless number that depends on the shape of the object.

Drag

- Cross-section areas for objects of different shape.

Example 6.7 Air Resistance Compared to Rolling Friction

**Example 6.7** Air resistance compared to rolling friction

The profile of a typical 1300 kg passenger car, as seen from the front, is 1.6 m wide and 1.4 m high. Aerodynamic body shaping gives a drag coefficient of 0.35. At what speed does the magnitude of the drag equal the magnitude of the rolling friction?

**MODEL** Model the car as a particle. Use the models of rolling friction and drag. Note that this is not a constant-force situation.
Example 6.7 Air Resistance Compared to Rolling Friction

**Example 6.7** Air resistance compared to rolling friction

**Solution** Drag is less than friction at low speeds, where air resistance is negligible. But drag increases as \( v \) increases, so there will be a speed at which the two forces are equal in size. Above this speed, drag is more important than rolling friction. There’s no motion and no acceleration in the vertical direction, so we can write the free-body diagram that:

\[ F_{\text{drag}} = mg \]

Then \( F_{\text{drag}} = \mu mg \). Equating friction and drag, we have

\[ \sqrt{\frac{2mg}{\mu}} = \mu mg \]

Solving for \( v \), we find

\[ v = \sqrt{\frac{2g}{\mu}} \]

where the value of \( \mu \) for rubber on concrete was taken from Table 6.1.

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**Example 6.10** Make sure the cargo doesn’t slide

**Example 6.10** Make sure the cargo doesn’t slide

A 100 kg box of dimensions 50 cm \( \times \) 50 cm \( \times \) 50 cm is in the back of a flatbed truck. The coefficients of friction between the box and the bed of the truck are \( \mu_k = 0.40 \) and \( \mu_s = 0.20 \). What is the maximum acceleration the truck can have without the box slipping?

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**Terminal Speed**

- The drag force from the air increases as an object falls and gains speed.
- If the object falls far enough, it will eventually reach a speed at which \( F_{\text{drag}} = F_G \).
- At this speed, the net force is zero, so the object falls at a constant speed, called the terminal speed \( v_{\text{term}} \):

\[ v_{\text{term}} = \sqrt{\frac{2mg}{\mu}} \]
EXAMPLE 6.10 Make sure the cargo doesn’t slide

MODEL
- Let the box, which we’ll model as a particle, be the object of interest.
- Only the truck exerts contact forces on the box.
- The box does not slip relative to the truck.
- If the truck bed were frictionless, the box would slide backward as seen in the truck’s reference frame as the truck accelerates.
- The force that prevents sliding is static friction.
- The box must accelerate forward with the truck: \( a_{\text{box}} = a_{\text{truck}} \).

Known
- \( m = 100 \text{ kg} \)
- Box dimensions: 50 cm \( \times \) 50 cm \( \times \) 50 cm
- \( \mu_s = 0.40 \)
- \( \mu_k = 0.20 \)

Find
- Acceleration at which box slips

\[
\sum F_x = m a_x = \sum F_y = 0
\]

\[
F_{\text{box}} = F_{\text{truck}} = m a_{\text{truck}} = m a_{\text{box}}
\]
Example 6.10 Make Sure the Cargo Doesn’t Slide

**EXAMPLE 6.10** Make sure the cargo doesn’t slide

Known
- m = 1000 kg
- Box dimensions: 50 cm x 50 cm x 50 cm
- \( \mu_s = 0.40 \), \( \mu_k = 0.20 \)

Find
- Acceleration at which box slips

The cargo is placed on a ramp and is at rest. The ramp has an angle \( \theta \) with the horizontal and has a friction coefficient \( \mu_k \). The ramp is inclined at an angle \( \theta \) with the horizontal.

**Solution:**

1. **Free Body Diagram:**
   - Gravity \( F_g \)
   - Normal force \( F_N \)
   - Friction force \( F_f \)

2. **Equations of Motion:**
   - **Horizontal Direction:***
     \[ F_f = m a_x \]
   - **Vertical Direction:**
     \[ m g - F_N = m a_y \]

3. **Force and Friction:**
   - Friction force:
     \[ F_f = \mu_k F_N \]
   - Normal force:
     \[ F_N = m g \cos \theta \]

4. **Solve for Acceleration:**
   - **Horizontal Direction:**
     \[ a_x = \frac{F_f}{m} = \mu_k m g \cos \theta \]
   - **Vertical Direction:**
     \[ a_y = \frac{m g - m g \cos \theta}{m} = g (1 - \cos \theta) \]

**Note:**
- The acceleration \( a_x \) and \( a_y \) are the components of the acceleration vector.
- The system is in equilibrium if \( a = 0 \).

**Chapter 6 Summary Slides**

**General Principles**

**Two Explanatory Models**
- An object on which there is no net force is in mechanical equilibrium.
  - Objects at rest.
  - Objects moving with constant velocity.
  - Newton’s second law applies with \( a = 0 \).

**A Problem-Solving Strategy**

1. **Translate words into symbols.**
2. **Draw a sketch to define the situation.**
3. **Draw a free-body diagram.**
4. **Identify forces.**
5. **Use Newton’s second law:**
   \[ \vec{F}_n = m \vec{a} \]
   - “Read” the vectors from the free-body diagram. Use kinematics to find velocities and positions.
6. **Assess** if the result is reasonable. Does it have correct units and significant figures?
**Important Concepts**

Specific information about three important descriptive models:

- **Gravity**  \( F_g = (mg, \text{ down}) \)
- **Friction**  
  
  \[ F_f = (\mu \text{n}, \text{ direction opposite the motion}) \]
- **Drag**  
  
  \[ F_{\text{drag}} = \left( \frac{1}{2} \rho AV^2, \text{ direction opposite the motion} \right) \]

**Important Concepts**

Newton’s laws are vector equations. You must write them out by components:

\[ (F_{\text{net}})_x = \sum F_x = ma_x \]

\[ (F_{\text{net}})_y = \sum F_y = ma_y \]

The acceleration is zero in equilibrium and also along an axis perpendicular to the motion.

**Applications**

**Mass** is an intrinsic property of an object that describes the object’s inertia and, broadly speaking, its quantity of matter.

**Weight** is an object’s reading on a spring scale when the object is at rest relative to the scale. Weight is the result of weighing. An object’s weight depends on its mass, its acceleration, and the strength of gravity. An object in free fall is weightless.

**Applications**

A falling object reaches a terminal speed:

\[ v_{\text{terminal}} = \sqrt{\frac{2mg}{C_dA}} \]

Terminal speed is reached when the drag force exactly balances the gravitational force: \( F = mg \).