A quantity that is fully described by a single number is called a **scalar quantity** (i.e., mass, temperature, volume).

A quantity having both a magnitude and a direction is called a **vector quantity**.

The geometric representation of a vector is an arrow with the tail of the arrow placed at the point where the measurement is made.

We label vectors by drawing a small arrow over the letter that represents the vector, i.e.,: $\vec{a}$ for position, $\vec{v}$ for velocity, $\vec{a}$ for acceleration.

- Vector addition is easily extended to more than two vectors.
- The figure shows the path of a hiker moving from initial position 0 to position 1, then 2, 3, and finally arriving at position 4.
- The four segments are described by displacement vectors $\vec{D}_1$, $\vec{D}_2$, $\vec{D}_3$, and $\vec{D}_4$.
- The hiker’s net displacement, an arrow from position 0 to 4, is $\vec{D}_{net} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3 + \vec{D}_4$.
- The vector sum is found by using the tip-to-tail method three times in succession.
Coordinate Systems and Vector Components

- A coordinate system is an artificially imposed grid that you place on a problem.
- You are free to choose:
  - Where to place the origin, and
  - How to orient the axes.
- Below is a conventional $xy$-coordinate system and the four quadrants I through IV.

Vector Components

⇒ Components of a vector are two perpendicular vectors that add together to produce the original vector.

Vector Components

⇒ A component of a vector is really just a projection of the vector onto an axis.

⇒ Any vector can be expressed as the sum of its components.
Component Vectors

- The figure shows a vector $\vec{A}$ and an $xy$-coordinate system that we’ve chosen.
- We can define two new vectors parallel to the axes that we call the component vectors of $\vec{A}$, such that $\vec{A} = \vec{A}_x + \vec{A}_y$.
- We have broken $\vec{A}$ into two perpendicular vectors that are parallel to the coordinate axes.
- This is called the decomposition of $\vec{A}$ into its component vectors.

Components

- Suppose a vector $\vec{A}$ has been decomposed into component vectors $\vec{A}_x$ and $\vec{A}_y$, parallel to the coordinate axes.
- We can describe each component vector with a single number called the component.
- The component tells us how big the component vector is, and, with its sign, which ends of the axis the component vector points toward.
- Shown to the right are two examples of determining the components of a vector.

Tactics: Determining the Components of a Vector

**TACTICS BOX 3.1**

**Determining the components of a vector**

- The absolute value $|A_x|$ of the $x$-component $A_x$ is the magnitude of the component vector $A_x$.
- The sign of $A_x$ is positive if $A_x$ points in the positive $x$-direction (right), negative if $A_x$ points in the negative $x$-direction (left).
- The $y$-component $A_y$ is determined similarly.

Moving Between the Geometric Representation and the Component Representation

- We will frequently need to decompose a vector into its components.
- We will also need to “reassemble” a vector from its components.
- The figure to the right shows how to move back and forth between the geometric and component representations of a vector.

The magnitude and direction of $\vec{A}$ are found from the components. In this example, $A = \sqrt{A_x^2 + A_y^2}$, $\theta = \tan^{-1}(A_y/A_x)$.

The components of $\vec{A}$ are found from the magnitude and direction.
Moving Between the Geometric Representation and the Component Representation

- If a component vector points left (or down), you must manually insert a minus sign in front of the component, as done for $B_y$ in the figure to the right.
- The role of sines and cosines can be reversed, depending upon which angle is used to define the direction.
- The angle used to define the direction is almost always between $0^\circ$ and $90^\circ$.

Unit Vectors

- Each vector in the figure to the right has a magnitude of 1, no units, and is parallel to a coordinate axis.
- A vector with these properties is called a unit vector.
- These unit vectors have the special symbols:
  \[ \hat{i} = (1, \text{positive } x\text{-direction}) \]
  \[ \hat{j} = (1, \text{positive } y\text{-direction}) \]
- Unit vectors establish the directions of the positive axes of the coordinate system.

Components of a Vector (using unit vector notation)

- The unit vectors have magnitude 1, no units, and point in the $+x$-direction and $+y$-direction.

\[ \mathbf{A} = A_x \hat{i} + A_y \hat{j} \]
Vector Algebra

- When decomposing a vector, unit vectors provide a useful way to write component vectors:
  \[
  \vec{A}_x = A_x \hat{i} \\
  \vec{A}_y = A_y \hat{j}
  \]
- The full decomposition of the vector \( \vec{A} \) can then be written
  \[
  \vec{A} = \vec{A}_x + \vec{A}_y = A_x \hat{i} + A_y \hat{j}
  \]

Working With Vectors

- We can perform vector addition by adding the \( x \)- and \( y \)-components separately.
- This method is called **algebraic addition**.
- For example, if \( \vec{D} = \vec{A} + \vec{B} + \vec{C} \), then
  \[
  D_x = A_x + B_x + C_x \\
  D_y = A_y + B_y + C_y
  \]
- Similarly, to find \( \vec{R} = \vec{P} - \vec{Q} \) we would compute
  \[
  R_x = P_x - Q_x \\
  R_y = P_y - Q_y
  \]
- To find \( \vec{T} = c \vec{S} \), where \( c \) is a scalar, we would compute
  \[
  T_x = cS_x \\
  T_y = cS_y
  \]
• For some problems it is convenient to tilt the axes of the coordinate system.
• The axes are still perpendicular to each other, but there is no requirement that the $x$-axis has to be horizontal.
• Tilted axes are useful if you need to determine component vectors “parallel to” and “perpendicular to” an arbitrary line or surface.
**Important Concepts**

A vector is a quantity described by both a magnitude and a direction.

- **Direction**
- The vector describes the situation at the point.
- The length or magnitude is denoted by a magnitude.

**Important Concepts**

Unit Vectors

Unit vectors have magnitude 1 and no units. Unit vectors \( \mathbf{i} \) and \( \mathbf{j} \) define the directions of the \( x \)- and \( y \)-axes.

**Using Vectors**

**Components**

- The component vectors are parallel to the \( x \) and \( y \) axes:
  - \( \mathbf{x} = x_i + x_j \)
  - \( \mathbf{y} = y_j \)

- In the figure, the right, for example:
  - \( x_i = 3 \) north
  - \( y_j = 4 \) west
  - \( \mathbf{A} = 3 \mathbf{i} + 4 \mathbf{j} \)
  - **Minor signs need to be included if the vector points down or left.**

**Using Vectors**

**Working Graphically**

- Addition:
  - \( \mathbf{A} + \mathbf{B} \)
- Negative:
  - \( -\mathbf{A} \)
- Subtraction:
  - \( \mathbf{A} - \mathbf{B} \)
- Multiplication:
  - \( \mathbf{A} \times \mathbf{B} \)
Using Vectors

**Working Algebraically**

Vector calculations are done component by component: \( \vec{C} = \vec{A} + \vec{B} \) means:
\[
\begin{align*}
C_x &= A_x + B_x \\
C_y &= A_y + B_y
\end{align*}
\]

The magnitude of \( \vec{C} \) is then \( C = \sqrt{C_x^2 + C_y^2} \) and its direction is found using \( \tan^{-1} \).