Chapter 5 — Systems of Linear Equations in Two-Variables

Section 5.1 — Solving Systems of Linear Equations by Graphing
Section 5.2 — The Substitution Method
Section 5.3 — The Addition/Elimination Method
Section 5.4 — Application Problems of Systems of Linear Equations

Answers
Section 5.1 Solving Systems of Linear Equations by Graphing

5.1 – Solving Systems of Linear Equations by Graphing Worksheet

Example:
Solve the system by graphing. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check. You must show a graph and state an answer for the problem.

a) \[ 4x + 2y = 8 \]
\[ 3x - y = 1 \]

To solve a system of linear equations by graphing, first graph each equation on the same set of axes.

First, graph the top equation. This will be the “blue” equation/line.

\[ 4x + 2y = 8 \]

There are different ways to graph this equation. One may graph by solving for \( y \) and using the slope and y-intercept, or graph by plotting points. It is written in standard form, so finding the intercepts is the easiest.

To find the x-intercept, plug in \( y = 0 \) and solve for \( x \):

\[ 4x = 8 \]
\[ x = 2 \]

This means the x-intercept is the point \( (2, 0) \).

To find the y-intercept, plug in \( x = 0 \) and solve for \( y \):

\[ 2y = 8 \]
\[ y = 4 \]

This means the y-intercept is the point \( (0, 4) \).

Next, graph the bottom equation. This will be the “red” equation/line.

\[ 3x - y = 1 \]

This equation is written in standard form, but the intercepts contain fraction-values. To graph, choose to solve for \( y \), in order to write in slope-intercept form.

\[ -y = -3x + 1 \]
\[ y = 3x - 1 \]

Subtract 3\( x \) from both sides of the equation.

Divide all terms on both sides by \(-1\). Now, the equation is in slope-intercept form.

For the “red” line, the y-intercept is \( (0, -1) \). The slope is \( m = -3 \).

Now, use the graph to find the solution to the system. Finding a solution to a system means finding an ordered pair (point) that satisfies BOTH equations.

In this case, the two lines intersect and that intersection point (which is the only point on BOTH lines, is the only solution to the system.

Description of the graph: Two lines intersect.

Characteristics of lines: The lines have different slopes—hence, they “cross”.

Type of system: Consistent

Answer: \( (1, 2) \) The point of intersection is the solution to the system.

“Blue” check:

\[ 4(1) + 2(2) \neq 8 \]
\[ 4 + 4 \neq 8 \]
\[ 8 = 8 \]

“Red” check:

\[ 3(1) - 2 \neq 1 \]
\[ 3 - 2 \neq 1 \]
\[ 1 = 1 \]
Homework

1. In your own words, describe how to check if a given point is a solution to a system of linear equations in two variables.

2. In your own words, describe how to use graphing to solve a system of linear equations in two variables.

3. What are the three different types of systems of linear equations in two variables?

4. When asked to solve a system of linear equations in two variables, what are the three appropriate types of answers?

5. How many solutions does a consistent system of linear equations in two variables have? Draw a sketch of an example of a graph of this type of system.

6. How many solutions does an inconsistent system of linear equations in two variables have? Draw a sketch of an example of a graph of this type of system.

7. How many solutions does a dependent system of linear equations in two variables have? Draw a sketch of an example of a graph of this type of system.

8. In your own words, describe how to determine without graphing if a system of linear equations in two variables is consistent.

9. In your own words, describe how to determine without graphing if a system of linear equations in two variables is inconsistent.

10. In your own words, describe how to determine without graphing if a system of linear equations in two variables is dependent.

Determine whether the given point is a solution to the system of linear equations.

11. \( y = 5x - 4 \)
    \( y = -2x - 2 \)
    \((2, 6)\)

12. \( x - \frac{1}{2}y = -5 \)
    \( 5x + 3y = -3 \)
    \((-3, 4)\)

13. \( 4x + 3 = y \)
    \( \frac{1}{2}y - 2 = x \)
    \((\frac{1}{2}, 5)\)

14. \( 2x - 6y = 13 \)
    \( x - 3y = 11 \)
    \((0, 8)\)

Determine the number of solutions each system graphed below has. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

15.
16.

17.

18.

19.
Solve each system by graphing. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check every other even problem. You must show a graph and state an answer for each problem.

20. \[ \begin{align*}
y &= -2x + 3 \\
y &= 2x - 1
\end{align*} \]

21. \[ \begin{align*}
y &= 3x - 2 \\
y &= -\frac{1}{2}x + 5
\end{align*} \]

22. \[ \begin{align*}
y &= -\frac{2}{3}x \\
y - 2x &= 8
\end{align*} \]

23. \[ \begin{align*}
-2x + 3y &= 12 \\
y &= \frac{2}{3}x - 1
\end{align*} \]

24. \[ \begin{align*}
y &= 3 \\
x - 2y &= -2
\end{align*} \]

25. \[ \begin{align*}
y &= \frac{1}{2}x - 1 \\
y - 3x &= 9
\end{align*} \]

26. \[ \begin{align*}
3x + y &= 5 \\
y &= -\frac{1}{2}x - 3
\end{align*} \]

27. \[ \begin{align*}
y &= -2x + 4 \\
6x + 3y &= 12
\end{align*} \]

28. \[ \begin{align*}
2x &= -4 \\
y &= x - 3
\end{align*} \]

29. \[ \begin{align*}
x + y &= 4 \\
y &= -x
\end{align*} \]

30. \[ \begin{align*}
y &= 6x + 3 \\
6x + 3y &= -15
\end{align*} \]
Section 5.2 The Substitution Method

Example:
Solve each system by the Substitution Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

a) \[ 4x + 2y = 8 \]
\[ 3x - y = 1 \]

First, isolate a variable (either \(x\) or \(y\)) in one of the equations. If one of the variables has a coefficient of 1 or \(-1\), choose that variable to isolate.

*For this system, the easiest variable to isolate is the \(y\) in the bottom equation.*

\[ -y = -3x + 1 \]

Subtract \(3x\) from both sides of the equation.

\[ y = 3x - 1 \]

Divide all terms on both sides by \(-1\). Now, \(y\) is isolated.

Next, plug in the expression you obtained by isolating into the OTHER equation for the corresponding variable.

*For this system, plug in \(3x - 1\) for the \(y\) in the top (other) equation.*

\[ 4x + 2(3x - 1) = 8 \]

Parentheses are used around the expression since it contains more than one term. Notice that after this substitution, the remaining equation only has one type of variable, in this case: \(y\). Solve for it.

\[ 4x + 6x - 2 = 8 \]
\[ 10x - 2 = 8 \]
\[ 10x = 10 \]
\[ x = 1 \]

Now, plug the value found in step 2, to ANY equation with both variables to find value of the other variable in the system.

*For this system, it’s easiest to plug 1 into the bottom equation for \(x\).*

\[ 3(1) - y = 1 \]
\[ 3 - y = 1 \]
\[ -y = -2 \]
\[ y = 2 \]

The solution to this system is \((1, 2)\).

If the system were graphed, the two lines would intersect at the point \((1, 2)\). This is a consistent system with one solution.

Homework

1. What are the three different types of systems of linear equations in two variables?

2. When asked to solve a system of linear equations in two variables, what are the three appropriate types of answers?

3. When asked to solve a system of linear equations in two variables by the Substitution Method, how do you choose which variable in which equation to isolate?

4. When there is no solution to a system of linear equations in two variables, what does the last step of the Substitution Method look like?

5. When there are an infinite number of solutions to a system of linear equations in two variables, what does the last step of the Substitution Method look like?
Solve each system by the Substitution Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check every other odd problem. You must show substitution and state an answer for each problem.

6. \[ y = x + 8 \]
   \[ 2x + 3y = 9 \]

7. \[ x = 11 - 3y \]
   \[ 3x - 5y = -23 \]

8. \[ 5x - 2y = 14 \]
   \[ y = -3x - 7 \]

9. \[ 2y + x = -3 \]
   \[ x = 13 - 2y \]

10. \[ x - 3y = -1 \]
    \[ -2x + 5y = -1 \]

11. \[ 5x - 2y = 39 \]
    \[ 3x + y = 19 \]

12. \[ 2x - 4y = 6 \]
    \[ -x + 2y = -3 \]

13. \[ y = 3x - 13 \]
    \[ y = \frac{1}{2}x + 2 \]

14. \[ 3x = 6y + 9 \]
    \[ -4x + 8y = -12 \]

15. \[ 3x - 5y = 45 \]
    \[ 6x - 2y = 66 \]

16. \[ \frac{3}{2}x + \frac{1}{2}y = \frac{3}{2} \]
    \[ \frac{1}{4}x + \frac{3}{4}y = -\frac{1}{4} \]

17. \[ \frac{1}{2}x - \frac{1}{4}y = 1 \]
    \[ \frac{10}{3}x - \frac{5}{3}y = 5 \]

18. \[ \frac{1}{2}x + y = 4 \]
    \[ 3x - 2y = 12 \]

19. \[ 8x - 3y = 220 \]
    \[ 4x + 12y = 920 \]

20. \[ 0.15x - 0.10y = 6 \]
    \[ 0.3x + 0.1y = 3 \]

21. \[ 11x + 22y = -121 \]
    \[ 36x - 40y = -60 \]
Section 5.3  The Addition/Elimination Method

Example:
Solve the system by the Addition/Elimination Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them.

a) \[
\frac{2}{3}x + \frac{1}{6}y = \frac{4}{3} \\
3x - 2y = 6
\]

Since the top equation in this system contains fractions, multiply by it by the LCD = 3; yielding:

\[
2x + y = 4 \\
3x - 2y = 6
\]

To solve by Addition/Elimination, first, choose which variable to eliminate. If one of the variables has a coefficient of 1 or \(-1\), choose that variable to eliminate.

For this system, the easiest variable to eliminate is \(y\).

Next, multiply one (or both) of the equations by the appropriate quantity, so that the coefficients of the variable to be eliminated are opposites.

For this system, to make the coefficients of \(y\) opposites, multiply the top equation by 2.

\[
4x + 2y = 8 \\
3x - 2y = 6
\]

Now, add the two equations together.

\[
\begin{align*}
4x + 2y &= 8 \\
3x - 2y &= 6
\end{align*}
\]

\[
7x = 14
\]

Solve for the remaining (non-eliminated) variable.

In this system, \(7x = 14\), so \(x = 2\).

Finally, plug the value found into ANY equation with both variables to find value of the other variable in the system.

For this system, it’s easiest to plug 2 into the top equation (without fractions) for \(x\).

\[
\begin{align*}
2(2) + y &= 4 \\
4 + y &= 4 \\
y &= 0
\end{align*}
\]

The solution to this system is \((2, 0)\).

If the system were graphed, the two lines would intersect at the point \((2, 0)\). This is a consistent system with one solution.

Homework

1. What are the three different types of systems of linear equations in two variables?
2. When asked to solve a system of linear equations in two variables, what are the three appropriate types of answers?
3. When asked to solve a system of linear equations in two variables by the Addition/Elimination Method, what is the first thing to check before you begin?
4. When asked to solve a system of linear equations in two variables by the Addition/Elimination Method, how do you choose which variable in which equation to eliminate?
5. When there is no solution to a system of linear equations in two variables, what does the last step of the Addition/Elimination Method look like?
6. When there are an infinite number of solutions to a system of linear equations in two variables, what does the last step of the Addition/Elimination Method look like?
Solve each system by the Addition/Elimination Method. If there is one solution, find it. If there is no solution, state so. If there are infinitely many solutions, describe them. Check every other odd problem. You must show addition/elimination and state an answer for each problem.

7. \[3x - y = 11\] 
   \[2x + y = -1\]

8. \[x + 2y = 17\]
   \[-x + 5y = 39\]

9. \[3x + 2y = 12\]
   \[4x - y = 5\]

10. \[5x - 3y = -12\]
    \[8 - 2y = 5x\]

11. \[11x = 3y + 29\]
    \[2x + 6y = 38\]

12. \[2x - 4y = 10\]
    \[2y = x + 3\]

13. \[10y = 52 - 8x\]
    \[4x = 26 - 5y\]

14. \[3x - 8y = -11\]
    \[2x + 6y = 38\]

15. \[22 + 10y = 6x\]
    \[8x + 15y = 1\]

16. \[2x - 8y = 6\]
    \[3x = 12y + 9\]

17. \[10x = 15y + 8\]
    \[8x - \frac{18}{5} = 6y\]

18. \[7x = 9 - 2y\]
    \[4y = 11 - 14x\]

19. \[\frac{1}{2} x - \frac{5}{6} y = 24\]
    \[\frac{3}{2} x + \frac{1}{8} y = 3\]

20. \[\frac{1}{3} x + \frac{2}{3} y = 8\]
    \[\frac{3}{8} x + \frac{1}{6} y = \frac{10}{3}\]

21. \[1.25x + 2.35y = 47.75\]
    \[2.55x + 4.70y = 96\]

22. \[2x = 6\]
    \[11x - 17 = 8y\]
Section 5.4 Application Problems of Systems of Linear Equations

5.4 – Application Problems of Systems of Linear Equations Worksheet

Example:
For the problem below, define variables, write two equations, solve the system, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate. You may use a chart to organize your information, but it isn’t necessary.

a) Natalie owns a coffee shop. In her shop there are many varieties of coffee. One, an Ethiopian coffee sells for $7 per pound, and a second, a Colombian coffee, sells for $4 per pound. She’s found that some of her customers like a blend of the Ethiopian coffee and Colombian coffee. How much of each type of coffee should she mix to get 12 pounds of a mixture that sells for $6 per pound? Note: the price per pound of the mix is given, NOT the total value.

First, figure out what you are asked to find and what you are given.

Find:
Number of pounds of Ethiopian coffee: \( x \)
Number of pounds of Colombian coffee: \( y \)

Given:
Price per pound of Ethiopian coffee: $7 per pound
Price per pound of Colombian coffee: $4 per pound
Price per pound of mixture of both coffees: $6 per pound
Number of pounds in the mix of both coffees: 12 pounds

Sometimes it is helpful to organize this information in a chart. Notice that all quantities in an individual column have the same units. Price per pound is a type of rate.

<table>
<thead>
<tr>
<th>Title</th>
<th>Price per pound</th>
<th>Individual Amount</th>
<th>Total Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethiopian</td>
<td>$7 per pound</td>
<td>( x )</td>
<td>7( x )</td>
</tr>
<tr>
<td>Colombian</td>
<td>$4 per pound</td>
<td>( y )</td>
<td>4( y )</td>
</tr>
<tr>
<td>Mix</td>
<td>$6 per pound</td>
<td>12 pounds</td>
<td>6(12)</td>
</tr>
</tbody>
</table>

Next, write two equations based on the non-rate columns.

Equation about the individual amounts:
number of pounds of Ethiopian coffee + number of pounds of Colombian coffee = number of pounds in the mix of both coffees
\[
x + y = 12
\]

Equation about total value:
total value of Ethiopian coffee + total value of Colombian coffee = total value in the mix of both coffees
\[
7x + 4y = 6(12)
\]

Now, solve the system. You may solve it by graphing, The Substitution Method, or The Addition/Elimination Method. The Addition/Elimination Method is shown below.

\[
x + y = 12
\]
\[
7x + 4y = 72
\]

\[
-4[x + y = 12] \rightarrow -4x - 4y = -48
\]
\[
+ 7x + 4y = 72 \rightarrow 3x = 24
\]
\[
3x = 24 \rightarrow x = 8
\]

Plugging in 8 for \( x \) into the top equation yields:
\[
8 + y = 12 \rightarrow y = 4
\]

Natalie should mix 8 pounds of French Roast and 4 pounds of Colombian.

Homework
1. In your own words, describe how you can recognize that an application problem may be modeled with a system of equations.
2. If you plan to use a chart to organize the information in this mixture-type of “system of linear equations application problem”, how many rows and columns will you need, and what will each be labeled?
For each problem below, define variables, write two equations, solve the system, and answer the question(s). Give your answer(s) in simplest form with the correct units when appropriate. You may use a chart to organize your information, but it isn’t necessary.

3. Doug made a watermelon and strawberry salad for a potluck. Strawberries cost $3.25 per pound and watermelon costs $0.50 per pound. If he paid $7.00 to make a 3-pound salad, how many pounds of strawberries, and how many pounds of watermelon did he buy?

4. Ana has a 2003 VW Golf Tdi that she usually runs on biodiesel. During a recent trip to her biofuel supplier, she purchased a mix of B-99 and B-80. B-99 costs $4.55 per gallon and B-80 costs $4.25 per gallon. If she bought a total of 12 gallons of fuel and paid $4.35 per gallon for the fuel, how many gallons of B-99 did she buy, and how many gallons of B-80 did she buy?

5. During a chemistry class, a student needs to mix hydrochloric acid solutions: one containing 15% of hydrogen chloride with one containing 25% hydrogen chloride. How much of each solution should she mix to get 1.5 liters of hydrochloric acid solution that has 18% hydrogen chloride? Give your answer accurate to 2 decimal places.

6. Alejandro invested $4000, part at 4% simple interest and the rest at 3% simple interest for a period of 1 year. If he received a total annual interest of $145 from both investments, how much did he invest at each rate?

7. Sodium hypochlorite solution is commonly known as bleach. Sahil has household bleach that is 2% sodium hypochlorite and chlorination bleach that is 12% sodium hypochlorite. He wants 125 mL of bleach-mix that contains 6.25 mL of sodium hypochlorite. How much household bleach and how much chlorination bleach should he use? Give your answer accurate to 1 decimal place.

8. At a fundraiser, Patrick bought drinks and raffle tickets for his family. Each drink cost $0.75 and each raffle ticket cost $1.20. If he purchased a total of 10 items and spent a total of $10.20, how many drinks, and how many raffle tickets did he buy?

9. Gerry owns a candy and nut shop. In the shop, there are both bulk and packaged candy and nuts. A customer comes into the shop and explains that he needs a large amount of a mixture of chocolate covered cherries and amaretto cordials. The customer explains that he plans to make individual packages of the mixture and give a package to each guest at a dinner party. The chocolate covered cherries sell for $7.50 per pound and the amaretto cordials sell for $6.00 per pound. How many pounds of amaretto cordials must be mixed with 12 pounds of chocolate covered cherries to obtain a mixture that sells for $6.50 per pound? How many pounds of mixture will there be? Note: the price per pound of the mix is given, not the total value of the mix. Also, only the number of pounds of chocolate covered cherries is given, not the number of pounds in the mix.

10. The label on a 12-ounce can of frozen concentrate Hawaiian Punch indicates that when the can of concentrate is mixed with 3 cans (36-ounces) of cold water the resulting mixture is 10% fruit juice. Find the percent of pure juice in the can of concentrate.