Math 18
Final Review/Extra Credit

This is due on the day of the final as an extra credit assignment. On another document, I will post answers to this. But, to get any extra credit, you have to show the work.

1) Find the derivatives of the following:
   a) $5x^2 + \frac{3}{x} + 2\sqrt{x} + 4$
   b) $e^{2x^3}$
   c) $(x + 1)^3 (x + 2)^4$
   d) $\frac{3x^3 + 6x}{2x - 1}$

2) Find the integrals of the following:
   a) $\int e^{3x+1} \, dx$
   b) $\int (5x^2 + \frac{2}{x} + \sqrt{x}) \, dx$
   c) $\int_0^5 (6e^{-3x}) \, dx$
   d) $\int_1^3 (4x^2 + \frac{1}{x}) \, dx$
   e) $\int \frac{2x + 1}{(x^2 + x)^5} \, dx$

3) Find the equation of the line tangent to $y = e^{3x} + 2x$ at the point (0,1).

4) I put $1000 into a bank account at 6% interest compounded continuously.
   a) How much money do I have at the end of 3 years?
   b) How long does it take my money to double?
   c) What is the rate of change of my account balance in $ per year after 5 years?

5) We have a company that produces gadgets. Assume we sell the gadgets for $450 per gadget. The cost for us to produce $x$ gadgets is given by $C(x) = 10000 + 3x^2$.
   a) Find the revenue function.
   b) Find the profit function.
   c) What is the marginal cost for selling 100 gadgets?
   d) What is the average cost for selling 100 gadgets?
   e) Find the break even point(s).
   f) How many items should we produce to make a maximum profit?
   g) What is the maximum profit?
6) Gas is leaking out of a small hole in a tank at a rate of \( r(t) = 10e^{-0.2t} \) pounds per square inch per second at time \( t \) in seconds. How much gas leaks out of the tank in the first minute?

7) The marginal cost to produce \( x \) items is given by \( MC = 3x^2 + 2x - 5 \). Fixed costs are $200. Find the total cost to produce 100 items.

8) Find \( f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx} \) for the following functions:
   \( a) f(x, y) = x^2 y^3 \)
   \( b) f(x, y) = x^2 + 2xy + 3y^2 \)

9) Find and classify the critical points of \( f(x,y) = x^3 + y^2 - 2x^2 + 3y - 8 \).

10) A company manufactures a quantity \( x \) of one item selling for $8 per item and a quantity \( y \) of another item selling for $12 per item. The cost of producing the items is given by \( c(x,y) = x^2 + 2y^2 + 8 \).
   \( a) \) Find the revenue function for the company as a function of \( x \) and \( y \).
   \( b) \) Find the profit function for the company as a function of \( x \) and \( y \).
   \( c) \) Find the maximum profit that can be made.